Bose-condensation through resonance decay

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Abstract

We show that a system described by an equation of state which contains a high number of degrees of freedom (resonances) can create a considerable amount of super-fluid (condensed) pions through the decay of short-lived resonances, if baryon number and entropy are large and the dense matter decouples from chemical equilibrium earlier than from thermal equilibrium. The system cools down faster in the presence of a condensate, an effect that may partially compensate the enhancement of the lifetime expected in the case of quark-gluon-plasma formation.

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The investigation of hot hadronic matter through heavy ion collisions has shown that an understanding of medium effects and final state interactions is of great importance for the interpretation of the particle spectra in order to distinguish new physical phenomena like the formation of quark-gluon plasma (QGP) from background phenomena caused by "conventional" physical effects. We refer here, e.g., to the strangeness enhancement, the J/Ψ suppression and the soft-pion puzzle, which can in principle be explained either by the formation of a QGP or from purely hadronic origin (for a recent review, see ref. [1]).

An interesting explanation for the soft pion puzzle was proposed in [2]. The authors argued that the baryonic and mesonic resonances may decouple from chemical equilibrium but still remain in thermal equilibrium. Since the resonance production rates fall drastically after their decoupling from chemical equilibrium, the remaining short-lived resonances would then rapidly decay into pions. As a consequence, the pions may acquire a non-zero chemical potential μ_π which leads to a softer $p_\perp$-spectrum.

A different possible explanation for the absence of local chemical equilibrium of pions in the final (decoupling) stage, which should not be confused with the scenario discussed in ref. [2], was proposed in refs.[3, 4] where it is assumed that the pions are initially created out of local chemical (and even thermal!) equilibrium and it turns out that the system never reaches complete local chemical equilibrium[3, 4].

In the present letter, we investigate the scenario of ref. [2], where it is assumed that the dense matter reaches local thermodynamic equilibrium in an early stage of the expansion. The authors of [2] considered only the case that the chemical potential stays below the pion mass. Moreover, they did not attempt to determine the value of the pion chemical potential in terms of the temperature and baryon number density at the point of decoupling from local chemical equilibrium. Below, we address the problem if and under what conditions the decay of short-lived resonances can lead to the formation of a pionic Bose condensate, and what happens to the condensate in the subsequent expansion until the system decouples from local thermal equilibrium and the final state particles are emitted. To do so, we explicitly consider the transition from a resonance gas to a gas of stable and long-lived particles for a hadronic equation of state which contains the known resonances up to masses of 2 GeV, imposing constraints from the conservation of energy-momentum,

baryon number and strangeness. In particular, we shall show that for a system rich in entropy and baryon number μ_π may reach the pion mass and, consequently, the pions may form a Bose condensate in the central region in relativistic heavy ion collisions.

For a hydrodynamically expanding system, the formation of such a condensate implies the presence of a superfluid component. The idea that the hadronic matter might be superfluid was put forward a long time ago in a somewhat different context [5]. This could explain in a natural way the presence of a coherent component in multiparticle production. The implications for pion interferometry measurements will be discussed in a separate paper [6]. We note that although the creation of a remarkable amount of condensed pions is interesting by itself, these effects could also be used to answer the question of whether pions and resonances are in chemical equilibrium before they decouple from thermal equilibrium. Because the final stage in the hadronic phase can have a significant influence on the particle spectrum, the question of whether the freeze-out occurs in chemical and thermal equilibrium, as assumed in [7, 8] or only in thermal equilibrium [2] is of great importance.

In order to investigate the conditions necessary to form a condensate, let us first consider a system in local thermodynamic (i.e., thermal *and* chemical) equilibrium. The medium is then completely described in terms of its equation of state (EOS) which can be expressed in the form

$$\varepsilon_{fl} = f(T_{fl}, \mu_{fl}^k) \tag{1}$$

where ε_{fl} is the energy-density of the fluid, T_{fl} the temperature, and the quantities μ_{fl}^k are the chemical potentials related to conserved charges Q^k (such as baryon number B and strangeness S). The index fl refers to quantities which describe the fluid.

During a heavy ion collision we expect under certain conditions the formation of a system of hot hadronic matter or a QGP described by eq. (1) which is in chemical and thermal equilibrium. The system then undergoes a phase of hydrodynamic expansion until it has become so dilute that the interactions are no longer strong enough to maintain local equi-

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librium. This final stage, when the collective behaviour terminates and particles decouple from the dense matter, is referred to as freeze-out.

The scenario of ref.[2] differs from most of the other hydrodynamic descriptions of heavy ion collisions in so far as in [2] the dense matter is assumed to go out of chemical equilibrium but remain in thermal equilibrium for some time. Or, to put it differently, there will be a "chemical freeze-out" independent of and prior to the "thermal freeze-out". As was mentioned above, the transition out of chemical equilibrium leads to the decay of short-lived resonances and the appearance of a non-zero chemical potential of the pions.

Below, we shall assume that the chemical freeze-out, just like the thermal freeze-out, occurs "instantaneously" (i.e., on a three-dimensional hypersurface characterized by a condition such as $T(x,t) = const.$). This seems a reasonable assumption since the relevant time scales are the decay times of the short-lived resonances which are on the order of ~ 1 fm/c. To illustrate the rapidness of such a transition, we consider the simplified case of a system of pions and rho-mesons in local thermal equilibrium that undergoes a longitudinal scaling [9] expansion. In addition to the decay $\rho \to \pi\pi$ we also take into account the inverse process $\pi\pi \to \rho + X$. The equations which describe the time evolution of the system are

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P}{\tau} \tag{2}$$

$$\frac{dn_\pi}{d\tau} = -\frac{n_\pi}{\tau} + 2\frac{n_\rho}{\tau_\rho} - 2\langle\sigma v\rangle n_\pi^2 \tag{3}$$

$$\frac{dn_\rho}{d\tau} = -\frac{n_\rho}{\tau} - \frac{n_\rho}{\tau_\rho} + \langle\sigma v\rangle n_\pi^2 \tag{4}$$

where $\varepsilon = \varepsilon_\rho + \varepsilon_\pi$ and $P = P_\rho + P_\pi$ are the energy density and pressure, n_i $(i = \rho, \pi)$ are the number densities and $\tau \equiv \sqrt{t^2 - z^2}$ is the longitudinal propertime coordinate. τ_ρ is the lifetime of the ρ, and $\langle\sigma v\rangle$ is the thermal average of the cross section for the process $\pi\pi \to \rho + X$. We have solved eqs.(2- 4) numerically under the assumption that at $\tau = \tau_0$ the π's and ρ's are in chemical equilbrium at a temperature T_0, and that the system remains in local thermal equilibrium during the expansion (note that the eqs.(3,4) imply that $(n_\pi + 2n_\rho)\tau = const.$). Fig. 1 shows the time evolution of the pion chemical potential μ_π for $\tau_0 = 1$ fm/c and $T_0 = 200$ MeV. It turns out that for reasonable values $\langle\sigma v\rangle \leq 100$ mb, contributions of the inverse process are negligible. It can be seen that μ_π approaches its final value very fast, after less than 1 fm/c. For larger values of τ_0, the

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transition is even faster. The figure also shows that the decay of rho-mesons alone is not sufficient to drive the pions into the condensate (for that, one needs to take into account the decay contributions of all the other short-lived resonances as well).

We now return to the description of the expanding hot and dense matter at the point of chemical freeze-out. It will be assumed that the fluid freezes out into "stable" π's , K's, nucleons, Λ's and the long living resonances ω's and η's. The term "stable" here means that the decay of the particle or resonance occurs only after the complete (thermal and chemical) freeze-out of the system. In particular, the contribution of ω's and η's to the pion chemical potential is zero. Nevertheless, their later decay leads to a non-thermal component of the pion spectrum which is determined by the decay kinematics [7] .

The system of chemically frozen out particles can be described by a chemical decoupling temperature $T^{ch.f.}$, chemical potentials for baryons and strange particles, μ_B and μ_S, and a pion chemical potential, μ_π, which describes the overpopulation of pions due to the decoupling of resonances. The chemical potentials are determined by the requirement that the energy density ε_{fl}, the baryonic density b_{fl} and the strangeness density $s_{fl} = 0$ of the fluid are equal to those of the system after chemical freeze-out. We obtain the following system of equations:

$$
\begin{aligned}
\varepsilon_{fl} &= \varepsilon_\pi^{therm}(\mu_\pi, T^{ch.f.}) + m_\pi n_\pi^{con}\Theta(m_\pi - \mu_\pi) \\
&\quad + \sum_{i=K,N,\Lambda,\omega,\eta,...} \varepsilon_i^{therm}(\mu_S, \mu_B, T^{ch.f.}) \tag{5}
\end{aligned}
$$

$$
b_{fl} = \sum_k B_k\, n_k^{therm}(\mu_S, \mu_B, \mu_\pi, T^{ch.f.}) \tag{6}
$$

$$
s_{fl} = \sum_k S_k\, n_k^{therm}(\mu_S, \mu_B, \mu_\pi, T^{ch.f.}) = 0 \tag{7}
$$

where

$$
\varepsilon_i^{therm}(\mu_S, \mu_B, \mu_\pi, T) = \frac{g_i}{2\pi^2}\int p^2 dp \frac{E_i}{exp\left(\frac{E_i - \tilde{\mu}_i}{T}\right) \pm 1} \tag{8}
$$

$$
n_i^{therm}(\mu_S, \mu_B, \mu_\pi, T) = \frac{g_i}{2\pi^2}\int p^2 dp \frac{1}{exp\left(\frac{E_i - \tilde{\mu}_i}{T}\right) \pm 1} \tag{9}
$$

are the thermal parts of the energy density and the number density of particle species i, and where we have used the notation

$$\tilde{\mu}_i = \sum_k Q_i^k \mu^k \tag{10}$$

for the chemical potentials. Note that on the r.h.s. of eq. (10) the charges Q^k include the pion number N_π which is a conserved quantity after chemical freeze-out.

In eq. (5) we have introduced a term n_π^{con} that describes the condensed component of pions which will appear if the chemical equilibrium for pions reaches the pion mass. In this case an overpopulation of pion states occurs and the remaining pions will be forced into the ground state, thereby forming a Bose condensate. The fluid then consists of a thermal component and a superfluid component. The latter moves with the rapidity of the fluid, whereas the thermal pions have on top of the fluid rapidity a thermal distribution. The resultant single inclusive distribution of pions emitted from a hydrodynamically expanding source

$$
\begin{aligned}
\frac{1}{p_\perp}\frac{dn}{dydp_\perp} = & \; \frac{g_\pi}{(2\pi)^3} \int_\sigma \frac{p_\mu^i d\sigma^\mu}{exp\left(\frac{p_\mu^i u^\mu - \tilde{\mu}_\pi}{T_{fl}}\right) - 1} \\
& + \int_\sigma n_\pi^{con} u_\mu d\sigma^\mu \delta(y - \tanh\left(u_{||}/u_0\right))\delta^2(\vec{p}_\perp - m_\pi \vec{u}_\perp)
\end{aligned}
\tag{11}
$$

where y and $p_\perp$ are the rapidity and transverse momentum of the emitted pion, u^μ is the four-velocity field and σ is the freeze-out hypersurface. The first term on the r.h.s. of (11) is the thermal contribution given by the relativistic invariant Cooper-Frye formula [10], and the second term is the condensate contribution.

The coupled system of nonlinear of equations (5)-(7) have to be solved in order to determine from the fluid variables ε_{fl}, n_{fl} and $s_{fl} = 0$ the quantities μ_S, μ_B and μ_π (for $\mu_\pi < m_\pi$), or μ_S, μ_B and n_π^{con} (for $\mu_\pi = m_\pi$) which describe the system after chemical freeze-out.

Let us now investigate under what circumstances the system will develope a superfluid component. To this end, we apply a hadronic EOS which contains all the important

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resonances and a treatment of compression effects, thus retaining the essential features of nuclear matter near the ground state [11]. In addition to the work presented in [11] we take into account the effects of finite baryonic and mesonic masses, use Fermi statistics for the baryons and include mesons up to masses of 2 GeV.

Fig. 2 shows the thermal and the condensate component of pions at chemical freeze-out, and the chemical potentials μ_π, μ_b and μ_s, as functions of the temperature $T^{ch.f.}$ that characterizes the decoupling from chemical equilibrium, for three different values of the baryon number density b. It can be seen how the baryonic and strange chemical potentials, μ_b and μ_s, decrease with increasing temperature $T^{ch.f.}$. Clearly, the strange chemical potential induced by the finite baryon number never exeeceds 150 MeV. As the amount of heavier resonances increases with temperature, the pion chemical potential μ_π grows with $T^{ch.f.}$. It becomes equal to the pion mass in the temperature region 170-200 MeV; the value of $T^{ch.f.}$ where $\mu_\pi = m_\pi$ depends on the baryonic density. This behaviour is also reflected in the dependence of the thermal and condensate densities of pions on the temperature and baryon number density at chemical freeze-out. It can be seen that at sufficiently high baryonic densities and temperatures the condensate contribution to the chemically decoupled pions becomes significantly large. It is noteworthy that the condensation effect is enhanced if the density of the strange particles has not yet reached its equilibrium value. In this case the remaining energy is distributed among the non strange resonances, and this leads to an enhanced production of particles in the condensate.

Finally, we need to discuss the question whether or not the condensate survives the time period between the chemical and the thermal freeze-out. This is of particular importance if the dense matter decouples from chemical equilibrium at temperatures $T^{ch.f.} \sim 170-200$ MeV which are considerably higher than those usually associated with the thermal freeze-out, $T^{th.f.} \sim m_\pi$. After decoupling from chemical equilibrium, the system of stable and long-lived particles continues to expand until it has cooled down to the temperature $T^{th.f.}$.

For a one-dimensional scaling expansion, the evolution of the system is determined by the equations

$$s\,\tau \;=\; \text{const.} \tag{12}$$

$$b\,\tau \;=\; \text{const.} \tag{13}$$

$$s_e\,\tau \;=\; \text{const.} \tag{14}$$

$$n_\pi\,\tau \;=\; \text{const.} \tag{15}$$

$$n_\alpha\,\tau \;=\; \text{const.} \tag{16}$$

which describe the conservation of entropy S_e, baryon number B, strangeness S and pion number N_π (s_e, b, s and n_π being the corresponding densities, respectively). The index α labels those of the stable particle species ($K, N, \Lambda, \omega, \eta, ...$) which also have decoupled from chemical equilibrium. For the gas of stable particles and long-lived resonances, the thermodynamic quantities are given by the expressions on the r.h.s. of eqs. (5) – (10), with $s_e = \sum_i (\varepsilon_i + P_i - \tilde{\mu}_i n_i)/T$. Energy density and number density of the pions consist of a thermal and a condensate component, $\varepsilon_\pi = \varepsilon_\pi^{therm} + m_\pi n_\pi^{con}$ and $n_\pi = n_\pi^{therm} + n_\pi^{con}$, respectively. Note that the condensate does not contribute to the entropy. Consequently, the cooling of the system – i.e., the function $T(\tau)$ – does not depend on the superfluid component. Fig. 3 shows the cooling curves $T(\tau)$ of a pion gas, for different values of the pion chemical potential. For comparison we have also included the results for an ideal gas of massless pions. It can be seen that the cooling rate increases with increasing pion chemical potential and becomes maximal in the presence of a condensate. This implies that Bose condensates may reduce the lifetime of the fireball.

In particular, we are interested in the time dependence of the fraction of pions in the condensate, $f_{con} \equiv n_\pi^{con}/(n_\pi^{therm} + n_\pi^{con})$. Eqs. (12,15) imply that for the fraction of thermal pions, $f_{therm} = 1 - f_{con}$,

$$f_{therm}^{th.f.} \;=\; f_{therm}^{ch.f.} \; \frac{\left(n_\pi^{therm}/s_e\right)\big|_{th.f.}}{(n_\pi^{therm}/s_e)\big|_{ch.f.}} \tag{17}$$

where the labels $th.f.$ and $ch.f.$ refer to thermal and chemical freeze-out as before. The condensate survives until thermal decoupling if the r.h.s. of eq.(17) remains below 1. For the two limiting cases of a non-relativistic and of a ultrarelativistic gas, $f_{con}(\tau)$ =const., i.e., the fraction of particles in the condensate does not change during the expansion. For

the pion gas, it turns out that in the temperature range between 200 MeV and 150 MeV, f_{therm} varies by less than 10%.

To summarize, we have shown that the excited hadronic matter created in ultrarelativistic heavy ion collisions could form a pion condensate if (i) the system is rich in baryon number and entropy density, (ii) the hadrons decouple from local chemical equilibrium earlier than from thermal equilibrium, as suggested in [2], and (iii) the loss of chemical equilibrium occurs at temperatures of ~ 180 MeV or higher. The effect depends on the chemical decoupling temperature, on the baryonic density and, rather sensitively, on the resonance contribution to the EOS.[1] The effect is increased if the chemical equilibrium for strange particles is not complete, i.e. if the amount of strangeness which is initially zero and starts to increase during the collision and expansion process has not yet reached its equilibrium value.

It is not the purpose of the present study to prove that such a decoupling process occurs. To do so would require a detailed knowledge of the density and temperature dependence of the inelastic and elastic cross sections of hadrons in the dense matter. Rather, we would like to point out here that, if this decoupling happens in a certain temperature region (like it was quoted in [2]), then a pion condensate should appear.

If a pion condensate is created in a heavy ion collision, it may be distinguishable from the case of a resonance gas in thermal and chemical equilibrium essentially by the following effects.

- It should lead to a coherent component which then would appear in the two-particle correlation functions of identical pions through a reduction of the intercept of the correlation function and the appearance of a two exponent behaviour of the correlation function [13].

- The condensate component is moving with the fluid velocity and has no additional thermal component which smears out the distribution. This might lead to character-

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[1]Indeed, we have checked that for the EOS discussed in ref.[12], which was obtained by joining a parametrization of lattice QCD data to a Hagedorn type resonance gas EOS, about 40% of the pions would turn out to be in the condensed phase at normal freeze-out temperatures of ~ 140 MeV.

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istic bumps and shoulders in the rapidity and transverse momentum distributions of pions[2]. One also can expect that the coherent component disappears if one considers particles of velocities that exceed the maximum fluid velocities.

- The lifetime of the fireball may be reduced. This could at least partially compensate the enhancement of the lifetime expected [15] in the case that quark-gluon-plasma is formed in the intial stage.

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[2]In ref.[14] it was suggested that at $\mu_\pi = m_\pi$ the *thermal* component may lead to a dip at $p_\perp = 0$ (cf. also refs. given in [14])

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FIGURE CAPTIONS

Figure 1: The build-up of a pion chemical potential in an expanding gas of pions and rho-mesons out of chemical equilibrium.

Figure 2: Left column: Dependence of the chemical potentials for baryons (dashed), strangeness (dotted) and pions (solid) on the chemical freeze-out temperature $T^{ch.f.}$, for three different values of the baryon number density b. Right column: The thermal (solid) and condensate (dashed) components of the pion number densities at chemical freeze-out.

Figure 3: The temperature evolution of a pion gas for different chemical potentials μ_π and condensate contributions. For comparison, the curve for an ideal gas of massless pions has been included as well.